

# Belief Propagation-based Cooperative Localization for Quasi-Synchronous UWSNs with Sound Propagation Speed Uncertainty

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**Abstract**—Underwater wireless sensor networks (UWSNs) provide a promising approach for ocean monitoring. Locating the sensor nodes (SNs) in UWSNs is critical for annotating data samples. However, the global positioning system (GPS) is not available underwater. The locations of SNs are usually interpreted from anchor nodes which are deployed at known locations on the seafloor or sea surface. Due to the characteristics of UWSNs such as node sparsity, network asynchrony, energy restriction and sound propagation speed uncertainty, to locate the SN accurately is challenging. In this paper, we propose a factor graph framework and a Gaussian message-based belief propagation algorithm for the joint localization and synchronization problem. Particularly, the proposed algorithm jointly estimates the SN’s position and clock offset as well as the sound propagation speed in a fully distributed fashion. Simulation results show that the proposed algorithm converges in a few iterations and achieves good performance.

**Index Terms**—Underwater networks, cooperative localization, quasi-synchronous, propagation speed uncertainty, belief propagation.

## I. INTRODUCTION

Nowadays, underwater wireless sensor networks (UWSNs) play a main role in ocean data collection, disaster prevention, undersea exploration and assisted navigation [1] [2]. Underwater acoustic localization is a vital requirement for UWSNs applications. Since global positioning system (GPS) is not available underwater, locating the sensor node (SN) in UWSNs encounters severe problems: a) time-synchronization is hard to maintain mainly due to the individual oscillators equipped in SNs; b) the speed of underwater acoustic waves changes with the environment, and increases the inaccuracy of the distance estimation; c) UWSNs are even more energy limited than other WSNs, since the underwater node are often powered by a limited battery supply, and it is costly to recharge.

In this work, we propose a belief propagation (BP) based cooperative localization scheme for UWSNs considering the challenges above. Cooperative localization using BP has been widely applied in terrestrial wireless networks [3], but there is little work for UWSNs [4]. One main reason could be the

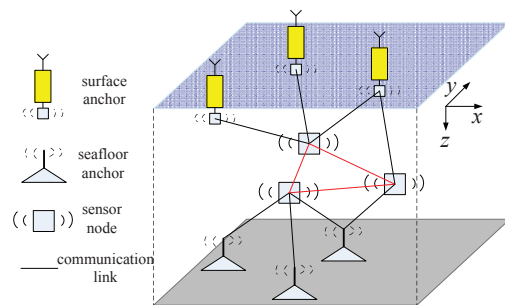


Fig. 1. A typical UWSN architecture.

high communication overhead needed by the belief exchanging among neighbouring nodes, especially when nonparametric BP is used. To reduce the communication overhead, we linearize the measurement model to represent all messages in the Gaussian form. Accordingly, only the mean and variance are required to be exchanged among neighbouring nodes. Hence, it significantly lowers the communication overhead.

We consider a typical UWSNs architecture as shown in Fig 1. There are three kinds of nodes in the network: surface anchors, seafloor anchors, and SNs. Surface anchors can obtain their positions through GPS and seafloor anchors are deployed at known positions. Therefore, these two kinds of anchors serve as "satellites" for the localization process of SNs. The benefit of cooperative localization is also depicted in Fig 1. Each SN cannot independently determine its own position based on distance measurements with respect to (w.r.t.) anchors. However, when the SNs can exchange messages mutually, they can help each other to achieve better localization performance in terms of both accuracy and coverage.

## II. SYSTEM MODEL

In practice, the underwater space is a 3D topology, but the SN can be equipped with a pressure sensor which can measure the depth. Thus, for the sake of exposition, we consider a 2D localization problem. Assume that an UWSNs

consists of  $M$  SNs and  $S$  anchors, and the position of node (SN or anchor)  $i$  is denoted as  $\mathbf{x}_i = [x_i, y_i]^T$ . We further define  $\mathcal{M}$  and  $\mathcal{S}$  represent the sets of SNs and anchors, respectively. The  $i$ th SN's neighboring SNs and anchors within its communication range are denoted by  $\mathcal{M}_i$  and  $\mathcal{S}_i$ . The set  $\Xi \triangleq \{(i, j) | i \in \mathcal{M}, j \in \{\mathcal{M}_i, \mathcal{S}_i\}\}$  contains all communication links. In quasi-synchronous UWSNs, there are only clock offsets between the anchors and the SNs. It is assumed that all anchors are synchronized with a reference time  $t$ . The local clock time of SN  $i$  can be modeled as

$$C_i(t) = t + \theta_i, \quad (1)$$

where  $\theta_i$  is the unknown clock offset. The SN  $i$  obtains a noisy time of arrival (TOA) measurement  $t_{ij}$  of the signal emitted by node  $j$

$$t_{ij} = \|\mathbf{x}_i - \mathbf{x}_j\|/c_{ij} + (\theta_i - \theta_j) + n_{ij}, \quad (2)$$

where  $c_{ij}$  represents the propagation speed of the communication link  $j \rightarrow i$ . The measurement noise is denoted by  $n_{ij}$ , and it is assumed to follow the Gaussian distribution with zero mean and variance  $\sigma_t^2$ . Collecting locations of all SNs into the set  $\chi \triangleq \{\mathbf{x}_i | i \in \mathcal{M}\}$ , and clock offsets into the set  $\Theta \triangleq \{\theta_i | i \in \mathcal{M}\}$ . The set  $\mathbf{T} \triangleq \{t_{ij} | (i, j) \in \Xi\}$  contains all TOAs measured by SNs, and the set  $\mathbf{C} \triangleq \{c_{ij} | (i, j) \in \Xi\}$  collects all the corresponding propagation speeds. Assume that the *a priori* distribution of node  $i$ 's position is  $p(\mathbf{x}_i)$  and that of the propagation speed  $c_{ij}$  is  $p(c_{ij})$ .

### III. BELIEF PROPAGATION-BASED COOPERATIVE LOCALIZER

For the Bayesian estimator, the goal is to estimate each SN's position and clock offset given all the TOA measurements and the *a priori* knowledge of the propagation speed uncertainty. According to the Bayesian rule, the joint *a posteriori* distribution  $p(\chi, \Theta, \mathbf{C} | \mathbf{T})$  can be written as

$$p(\chi, \Theta, \mathbf{C} | \mathbf{T}) \propto p(\mathbf{T} | \chi, \Theta, \mathbf{C}) p(\chi) p(\Theta) p(\mathbf{C}) \quad (3)$$

$$\propto \left( \prod_{(i,j) \in \Xi} p(t_{ij} | \mathbf{x}_i, \mathbf{x}_j, \theta_i, \theta_j, c_{ij}) p(c_{ij}) \right) \prod_{i \in \mathcal{M}} p(\mathbf{x}_i) \prod_{i \in \mathcal{M}} p(\theta_i),$$

where the *a priori* distributions of anchor's parameters are omitted due to the perfect knowledge of them, and the likelihood function can be written in the Gaussian form as,

$$p(t_{ij} | \mathbf{x}_i, \mathbf{x}_j, \theta_i, \theta_j, c_{ij}) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp \left\{ -\frac{(t_{ij} - \|\mathbf{x}_i - \mathbf{x}_j\|/c_{ij} - (\theta_i - \theta_j))^2}{2\sigma_t^2} \right\}. \quad (4)$$

As a result, the marginal distribution of the parameter concerned by SN  $i$ , denoted by  $\xi_i \in \{x_i, y_i, \theta_i, c_{ij}\}$  can be obtained by marginalization as

$$p(\xi_i | \mathbf{T}) = \int p(\chi, \Theta, \mathbf{C} | \mathbf{T}) d \sim \{\xi_i\}, \quad (5)$$

where notation " $\sim \{\xi_i\}$ " denotes all the parameters collected in  $\{\chi, \Theta, \mathbf{C}\}$  except  $\xi_i$ . Based on the marginal distribution,

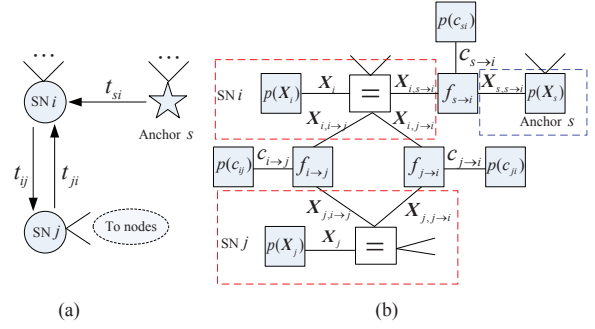


Fig. 2. An illustration of the factor graph: (a) the network topology; (b) the factor graph model of (a), where the vector  $\mathbf{X}_i = [\mathbf{x}_i^T, \theta_i]^T$ ,  $i \in \{\mathcal{M}, \mathcal{S}\}$ ,  $p(\cdot)$  represents the *a priori* distribution, and  $f_{i \rightarrow j}$ ,  $f_{j \rightarrow i}$  and  $f_{s \rightarrow i}$  are the likelihood functions. All the variables on edges connected to the equality node ( $\circ$ ) are the same variable.

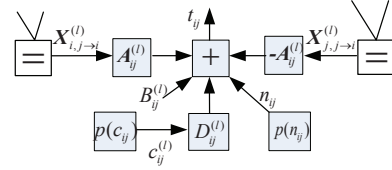


Fig. 3. The opened factor node  $f_{j \rightarrow i}$  after linearization

one can estimate the parameter  $\xi_i$  according to the MMSE (or MAP) criteria as  $\hat{\xi}_i = \int \xi_i p(\xi_i | \mathbf{T}) d\xi_i$ . However, it is intractable to calculate the integral directly especially in large-scale networks. In the following, we use a factor graph to represent the factorization of the joint *a posteriori* distribution in (3) and accordingly the marginal functions can be computed using the sum-product algorithm (SPA) on factor graphs [5].

As shown in Fig. 2, a sub-factor graph is provided to show the factor graph representation and the BP. In the factor graph, variables are denoted by edges and local functions are denoted by squares as factor nodes. There are two kinds of messages have to be calculated when performing the BP, i.e., the message from the factor node to the variable edges, and that from the variable edges to factor node. In order to obtain Gaussian-form expressions of all the messages, at the  $(l+1)$ th iteration, we linearize the TOA measurement function according to the first order Taylor expansion around the estimations at the  $l$ -th iteration, which is given by

$$t_{ij} = B_{ij}^{(l)} + \mathbf{A}_{ij}^{(l)} (\mathbf{X}_i - \mathbf{X}_j) + D_{ij}^{(l)} c_{ij} + n_{ij}, \quad (6)$$

where  $d_{ij}^{(l)} = \|\mathbf{x}_i^{(l)} - \mathbf{x}_j^{(l)}\|$ ,  $\mathbf{A}_{ij}^{(l)} = [\frac{(\mathbf{x}_i^{(l)} - \mathbf{x}_j^{(l)})^T}{c_{ij}^{(l)} d_{ij}^{(l)}}, 1]$ ,  $D_{ji}^{(l)} = -\frac{d_{ji}^{(l)}}{(c_{ji}^{(l)})^2}$  and  $B_{ij}^{(l)} = \frac{d_{ij}^{(l)}}{c_{ij}^{(l)}} + \frac{(\mathbf{x}_i^{(l)} - \mathbf{x}_j^{(l)})^T}{c_{ij}^{(l)} d_{ij}^{(l)}} (\mathbf{x}_j^{(l)} - \mathbf{x}_i^{(l)}) - D_{ij}^{(l)} c_{ij}^{(l)}$ . Based on the linearization in (6), we can open the factor node  $f_{j \rightarrow i}$  (the same can be done for  $f_{i \rightarrow j}$  and  $f_{s \rightarrow i}$ ) as shown in Fig. 3. According to the message-passing approach for the linear Gaussian systems [6], SN  $i$  can compute the incoming messages and outgoing messages in a convenient way. Assuming the message  $u_{\mathbf{X}_{j \rightarrow i}^{(l)} \rightarrow \mathbf{A}_{ij}^{(l)}} \sim$

$\mathcal{N}(\mathbf{X}_{j,j \rightarrow i}^{(l)}, \mathbf{m}_{\mathbf{X}_{j,j \rightarrow i}^{(l)} \rightarrow -\mathbf{A}_{ij}^{(l)}}, \mathbf{V}_{\mathbf{X}_{j,j \rightarrow i}^{(l)} \rightarrow -\mathbf{A}_{ij}^{(l)}})$ , where the first term in  $\mathcal{N}(\cdot)$  denotes the corresponding random variable that follows its distribution. For simplicity, we assume the *priori* distributions of  $\mathbf{X}_i$ ,  $i \in \mathcal{M}$  and  $c_{ij}$ ,  $(i, j) \in \Xi$  as  $\mathcal{N}(\mathbf{X}_i, \mathbf{m}_{\mathbf{X}_i}, \mathbf{V}_{\mathbf{X}_i})$  and  $\mathcal{N}(c_{ij}, c_0, \sigma_c^2)$ , respectively. Then the incoming message at the  $(l+1)$ th iteration can be written as

$$u_{\mathbf{X}_{i,j \rightarrow i}^{(l)} \rightarrow} \sim \mathcal{N}(\mathbf{X}_{i,j \rightarrow i}^{(l)}, \mathbf{m}_{\mathbf{X}_{i,j \rightarrow i}^{(l)} \rightarrow}, \mathbf{V}_{\mathbf{X}_{i,j \rightarrow i}^{(l)} \rightarrow}), \quad (7)$$

where

$$\begin{aligned} \mathbf{m}_{\mathbf{X}_{i,j \rightarrow i}^{(l)} \rightarrow} &= ((\mathbf{A}_{ij}^{(l)})^H \mathbf{A}_{ij}^{(l)})^\dagger (\mathbf{A}_{ij}^{(l)})^H \\ &\quad (t_{ij} - B_{ij}^{(l)} + \mathbf{A}_{ij}^{(l)} \mathbf{m}_{\mathbf{X}_{j,j \rightarrow i}^{(l)} \rightarrow -\mathbf{A}_{ij}^{(l)}} - D_{ij}^{(l)} c_0), \end{aligned} \quad (8)$$

$$\begin{aligned} \mathbf{V}_{\mathbf{X}_{i,j \rightarrow i}^{(l)} \rightarrow} &= ((\mathbf{A}_{ij}^{(l)})^H \mathbf{A}_{ij}^{(l)})^\dagger \\ &\quad (\mathbf{A}_{ij}^{(l)} \mathbf{V}_{\mathbf{X}_{j,j \rightarrow i}^{(l)} \rightarrow -\mathbf{A}_{ij}^{(l)}} (\mathbf{A}_{ij}^{(l)})^H + (D_{ij}^{(l)})^2 \sigma_c^2 + \sigma_t^2). \end{aligned} \quad (9)$$

Using the SPA rule, the outgoing message from SN  $i$  to SN  $j$  at the  $l$ -th iteration can be calculated as

$$u_{\Rightarrow \mathbf{X}_{i,i \rightarrow j}^{(l)}} = u_{\mathbf{X}_i \Rightarrow} = \prod_{k \in \{\mathcal{M}_i, \mathcal{S}_i\} \setminus j} u_{\mathbf{X}_{i,k \rightarrow i}^{(l)}}. \quad (10)$$

According to the property that the product of  $K$  Gaussian distribution is still Gaussian, which can be expressed as:

$$\prod_{k=1}^K \mathcal{N}(\mathbf{X}, \mathbf{m}_k, \mathbf{V}_k) \sim \mathcal{N}(\mathbf{X}, \mathbf{m}, \mathbf{V}), \quad (11)$$

where  $\mathbf{m} = \mathbf{V} \sum_{k=1}^K \mathbf{V}_k^\dagger \mathbf{m}_k$ ,  $\mathbf{V} = (\sum_{k=1}^K \mathbf{V}_k^\dagger)^\dagger$ , (10) follows the Gaussian distribution with the mean and variance as

$$\begin{aligned} \mathbf{m}_{\Rightarrow \mathbf{X}_{i,i \rightarrow j}^{(l)}} &= \mathbf{V}_{\Rightarrow \mathbf{X}_{i,i \rightarrow j}^{(l)}} (\mathbf{V}_{\mathbf{X}_i \Rightarrow}^\dagger \mathbf{m}_{\mathbf{X}_i \Rightarrow} \\ &\quad + \sum_{k \in \{\mathcal{M}_i, \mathcal{S}_i\} \setminus j} \mathbf{V}_{\mathbf{X}_{i,k \rightarrow i}^{(l)} \Rightarrow}^\dagger \mathbf{m}_{\mathbf{X}_{i,k \rightarrow i}^{(l)} \Rightarrow}), \end{aligned} \quad (12)$$

$$\mathbf{V}_{\Rightarrow \mathbf{X}_{i,i \rightarrow j}^{(l)}} = (\mathbf{V}_{\mathbf{X}_i \Rightarrow}^\dagger + \sum_{k \in \{\mathcal{M}_i, \mathcal{S}_i\} \setminus j} \mathbf{V}_{\mathbf{X}_{i,k \rightarrow i}^{(l)} \Rightarrow}^\dagger)^\dagger. \quad (13)$$

Based on all the above messages, we can calculate the SN  $i$ 's position belief at the  $(l+1)$ -th iteration as

$$\begin{aligned} b(\mathbf{X}_i^{(l+1)}) &= u_{\mathbf{X}_i \Rightarrow} = \prod_{k \in \{\mathcal{M}_i, \mathcal{S}_i\}} u_{\mathbf{X}_{i,k \rightarrow i}^{(l)}} \\ &\sim \mathcal{N}(\mathbf{X}_i^{(l+1)}, \mathbf{m}_{\mathbf{X}_i^{(l+1)}}, \mathbf{V}_{\mathbf{X}_i^{(l+1)}}), \end{aligned} \quad (14)$$

where

$$\begin{aligned} \mathbf{m}_{\mathbf{X}_i^{(l+1)}} &= \mathbf{V}_{\mathbf{X}_i^{(l+1)}} (\mathbf{V}_{\mathbf{X}_i \Rightarrow}^\dagger \mathbf{m}_{\mathbf{X}_i \Rightarrow} \\ &\quad + \sum_{k \in \{\mathcal{M}_i, \mathcal{S}_i\}} \mathbf{V}_{\mathbf{X}_{i,k \rightarrow i}^{(l)} \Rightarrow}^\dagger \mathbf{m}_{\mathbf{X}_{i,k \rightarrow i}^{(l)} \Rightarrow}), \end{aligned} \quad (15)$$

$$\mathbf{V}_{\mathbf{X}_i^{(l+1)}} = (\mathbf{V}_{\mathbf{X}_i \Rightarrow}^\dagger + \sum_{k \in \{\mathcal{M}_i, \mathcal{S}_i\}} \mathbf{V}_{\mathbf{X}_{i,k \rightarrow i}^{(l)} \Rightarrow}^\dagger)^\dagger. \quad (16)$$

Similarly, we can compute the incoming and outgoing message of the variable  $c_{ij}^{(l)}$ . The incoming message from the factor node  $D_{ij}^{(l)}$  to the variable edge  $c_{ij}^{(l)}$  at the  $(l+1)$ -th iteration is given by

$$u_{D_{ij}^{(l)} \rightarrow c_{ij}^{(l)}} \sim \mathcal{N}(c_{ij}^{(l)}, m_{D_{ij}^{(l)} \rightarrow c_{ij}^{(l)}}, V_{D_{ij}^{(l)} \rightarrow c_{ij}^{(l)}}), \quad (17)$$

where

$$m_{D_{ij}^{(l)} \rightarrow c_{ij}^{(l)}} = V_{D_{ij}^{(l)} \rightarrow c_{ij}^{(l)}} D_{ij}^{(l)} m_{\rightarrow D_{ij}^{(l)}} / V_{\rightarrow D_{ij}^{(l)}}, \quad (18)$$

$$V_{D_{ij}^{(l)} \rightarrow c_{ij}^{(l)}} = V_{\rightarrow D_{ij}^{(l)}} / (D_{ij}^{(l)})^2, \quad (19)$$

$$m_{\rightarrow D_{ij}^{(l)}} = t_{ij} - \mathbf{A}_{ij}^{(l)} (m_{\Rightarrow \mathbf{X}_{i,j \rightarrow i}^{(l)}} + m_{\Rightarrow \mathbf{X}_{j,i \rightarrow j}^{(l)}}) - B_{ij}^{(l)}, \quad (20)$$

$$V_{\rightarrow D_{ij}^{(l)}} = \mathbf{A}_{ij}^{(l)} (\mathbf{V}_{\Rightarrow \mathbf{X}_{i,j \rightarrow i}^{(l)}} + \mathbf{V}_{\Rightarrow \mathbf{X}_{j,i \rightarrow j}^{(l)}}) (\mathbf{A}_{ij}^{(l)})^H + \sigma_t^2. \quad (21)$$

The outgoing message from the variable edge  $c_{ij}^{(l)}$  to the factor node  $D_{ij}^{(l)}$  is the *priori* information of  $c_{ij}$ , i.e.  $u_{c_{ij}^{(l)} \rightarrow D_{ij}^{(l)}} = p(c_{ij})$ .

After obtaining the incoming message to the variable edge  $c_{ij}^{(l)}$ , we can calculate its belief at the  $(l+1)$ -th iteration as

$$\begin{aligned} b(c_{ij}^{(l+1)}) &= p(c_{ij}) u_{D_{ij}^{(l)} \rightarrow c_{ij}^{(l)}} \\ &\sim \mathcal{N}(c_{ij}^{(l+1)}, m_{c_{ij}^{(l+1)}}, V_{c_{ij}^{(l+1)}}), \end{aligned} \quad (22)$$

where

$$m_{c_{ij}^{(l+1)}} = V_{c_{ij}^{(l+1)}} (c_0 / \sigma_c^2 + m_{D_{ij}^{(l)} \rightarrow c_{ij}^{(l)}} / V_{D_{ij}^{(l)} \rightarrow c_{ij}^{(l)}}), \quad (23)$$

$$V_{c_{ij}^{(l+1)}} = (1 / \sigma_c^2 + 1 / V_{D_{ij}^{(l)} \rightarrow c_{ij}^{(l)}}). \quad (24)$$

For each SN, the  $(l+1)$ -th iteration ends after all the incoming messages are received and its belief is updated. Note that the outgoing messages from the same variable node to different factor nodes only differ in one term, Thus, we can broadcast the variable node's belief instead of transmitting the exact outgoing messages to different factor nodes in order to reduce the communication overhead. The whole self-localization process is described in Algorithm 1. Note that Algorithm 1 only describes the cooperative localization process of the SN's position and clock parameters while the sound propagation speed can be estimated in a similar way.

#### IV. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed algorithm through numerical simulations. We consider a  $5 \times 5$  km<sup>2</sup> plane with 13 static anchor nodes and  $M$  SNs, as shown in Fig 4. Anchors denoted by "□" are synchronized and have perfect knowledge of their positions. SNs denoted by "+" are randomly distributed on the plane. The prior distributions of SNs' position are assumed Gaussian with variances  $\sigma_{x_i}^2 = \sigma_{y_i}^2 = 100$  m<sup>2</sup>. Each SN has an clock offset

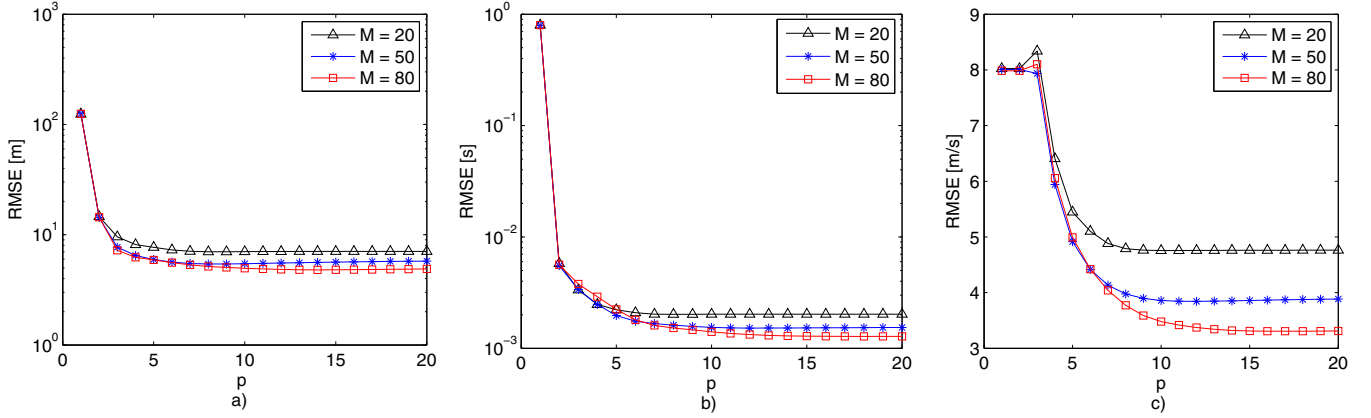


Fig. 5. RMSE of a) location, b) clock offset, c) sound propagation speed versus message passing iteration index  $p$  for different SN densities.

### Algorithm 1 BP-based cooperative localization algorithm

- 1: Nodes  $i \in \{\mathcal{M}, \mathcal{S}\}$  **in parallel**
- 2: initialize Gaussian variable prior
- 3:  $p(\mathbf{X}_i) \sim \mathcal{N}(\mathbf{X}_i, \mathbf{m}_{\mathbf{X}_i}, \mathbf{V}_{\mathbf{X}_i})$ ;
- 4:  $p(c_{ij}) \sim \mathcal{N}(c_{ij}, c_0, \sigma_c^2)$ ,  $(i, j) \in \Xi$ .
- 5: **end parallel**
- 6: **for**  $l = 1$  **to**  $N_{iter}$  (iteration index,  $N_{iter}$  is the maximum iteration)
- 7: nodes  $i \in \mathcal{M}$  **in parallel**
- 8: computing all the incoming messages as (7) by (8) and (9);
- 9: updating the belief as (14) by (15) and (16);
- 10: broadcasting the belief as outgoing messages;
- 11: **end parallel**
- 12: **end for**
- 13: calculate the SN's position and clock parameters by using MMSE or MAP estimator.

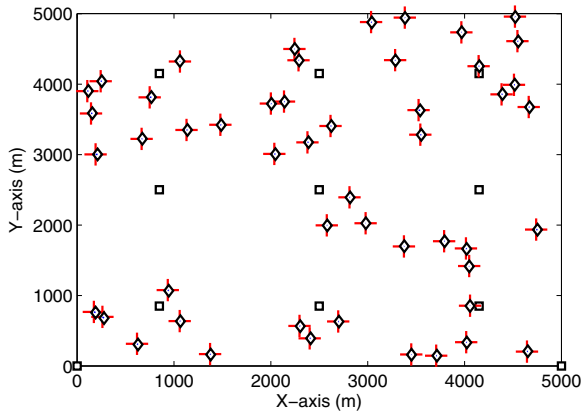


Fig. 4. Single trail localization results of the proposed algorithm.

which is drawn from a uniform distribution on the interval  $[-1 \text{ s}, 1 \text{ s}]$ . The maximum communication range is set to 2 km. Range measurements are performed by SNs. Communications between the neighboring nodes that are located within each

other's communication range are allowed. The *priori* probability of sound propagation speed in each communication link is Gaussian distribution with mean  $c_0 = 1500 \text{ m/s}$  and variance  $\sigma_c^2 = 10^2 \text{ (m/s)}^2$ . The standard deviation of range measurement errors is set to  $\sigma_d^2 = c_0^2 \sigma_t^2 = 1 \text{ m}^2$ .

Fig 4 shows a single trial performance of the proposed Algorithm 1 and the localization results are denoted by " $\diamond$ ". The maximum iteration  $N_{iter}$  is set to 10. It is seen that the position estimates are very close to the true positions. To further illustrate the performance of the proposed method, the root mean square errors (RMSEs) for the estimated parameters versus the number of iterations are evaluated. The RMSE is defined as  $\sqrt{1/(MN_{\text{exp}}) \sum_{j=1}^{N_{\text{exp}}} \sum_{i=1}^M \|\hat{\pi}_i^{(j)} - \pi_i^{(j)}\|^2}$ , where  $\hat{\pi}_i^{(j)}$  is the estimate ( $\hat{\mathbf{x}}_i$ ,  $\hat{\theta}_i$  or  $\hat{c}_{i,i'}$ ,  $(i, i') \in \Xi$ ) of the  $i$ -th SN in the  $j$ -th trial and  $N_{\text{exp}} = 100$ . In Fig 5, we show the RMSEs of the location estimation, clock offset estimation, and sound propagation estimation under different SN densities. As shown in the figure, the estimator converges within 10 iterations. Further, the localization performance improves as the SN density increases. This is because that each SN in dense network benefits from the sufficient information provided by its neighbours. However, the improvement becomes marginal as the SN density is large enough. Alternatively, one can increase the communication range to obtain a dense network, and similar RMSE results can be observed, which is not given here due to its similarity.

### V. CONCLUSION

In this paper, we studied the cooperative localization problem for a static quasi-synchronous UWSNs with sound propagation speed uncertainties. We propose a distributed cooperative localization algorithm to jointly estimate the SN's locations, clock offsets, and sound propagation speeds in all communication links. This algorithm represents the joint estimation problem on a factor graph and utilizes BP message passing to calculate the marginal probability density of the unknown parameters. Unlike the conventional particle-based message representing method, we linearize the nonlinear observation function using the Taylor expansion and represent

all the messages in Gaussian forms. As a result, only means and variances are required to be updated and transmitted. This is of great importance for BP-based cooperative localization underwater due to the severe constraints in bandwidth and energy. Simulation results show that the proposed algorithm converges in a few iterations and its performance increases as the network density increases. Our future work will focus on extension of this algorithm to dynamic UWSNs and considering the uncertainty of anchor position.

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