Underwater acoustic localization with uncertainties in propagation speed and time synchronization

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***Abstract*—Underwater acoustic localization is important for supporting underwater sensor networks. However, the hostile underwater environment makes it a very challenging mission. In this paper, we take uncertainties in sound propagation speed and time synchronization into account and propose a localization method. All anchors with known positions are synchronized, while all agents that need to perform localization are not synchronized with the anchors. The anchors measure the time of arrivals (ToAs) of the signals from the other anchors to estimate the sound propagation speed first. The agents measure the ToAs of the signals broadcast by the anchors, and combine the ToAs measured in two consecutive intervals to estimate the clock skews. After that the weight least squares (WLS) algorithm is used to calculate the agents' positions and clock offsets. Finally, the performance of the estimators of the clock skew, the clock offset, and the coordinates are refined via an alternative iteration process. The performance of the proposed estimators are evaluated through simulations.**
***Keywords:* localization, sound propagation speed, synchronization.**

I. Introduction

As an important service for underwater sensor networks (UWSNs), localization has been applied to numerous underwater sensor network applications, such as environment monitoring, ocean resource surveys and underwater security. However, the underwater hostile acoustic environment poses several challenges for localization in UWSNs. An error will be introduced if a classical constant (sound propagation speed), such as 1500 m/s, is used in all the periods of localization. Because the sound propagation speed may change with the time due to the characteristics of underwater environment vary temporally and spatially. Moreover, the long propagation delay and inherent mobility of underwater nodes make underwater clock synchronization still an issue to be solved. In addition, the power constraints of nodes limit the lifecycle of UWSNs. Therefore, it is necessary to propose an underwater acoustic localization algorithm with uncertainties in sound propagation speed and time synchronization taken into account.

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Among the typical range measurement methods, measuring time-of-arrival (ToA) or time-difference-of-arrival (TDoA) are the most potential approaches for underwater ranging [1]. Cheng et al. [2] proposed a silent localization schemes through measuring TDoAs of sequentially generating packets from anchors. The localization period of [2] is relative too long, and a known sound propagation speed is assumed, which will decrease the localization accuracy [3]. Kim et al. [4] suggested to estimate the sound propagation speed by packet exchange between anchors. The sound propagation speed is estimated by combining the channel characteristics and a sound propagation speed model in [5]. Moreover, Diamant et al. [6] proposed an algorithm to jointly estimate the agent location and the sound propagation speed. These proposed methods require multiple packet exchanges between anchors and agents, Thus it is not energy efficient for agents.

In order to overcome the disadvantages mentioned above, in this paper we take energy efficiency and uncertainties in sound propagation speed and time synchronization into consideration and design a localization algorithm. The main idea of our localization algorithm is as follows. The anchors broadcast signals and measure the ToAs of the signals from the other anchors to estimate the sound propagation speed first. The agent measures the ToAs of the signals from the anchors, and combines the ToAs measured in two consecutive intervals to estimate its clock skew. After that the weight least squares (WLS) algorithm is used to calculate the agent's position and clock offset. Finally, the alternative iteration process is used to refine the performance of the estimators of the clock skew, the clock offset, and the coordinates. Our method does not require the agents to transmit any signal. The agents localize themselves according to the received signals. Therefore, it is energy-efficient for the agent.

The remaining part of this paper is organized as follows. In Section II, we describe the scenario and the problem. A detailed elaboration on the estimate algorithm appears in Section III. The unbiased characteristics of our estimators and the CRBs of the sound speed, the clock skew, the clock offset and the agent's position are provided in Section IV. In

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Section V, the performance of our algorithm is evaluated via simulation. We conclude this paper in Section VI.

II. Problem Description

We would like to locate the underwater agents with the help of the floating anchors. A set of n anchors with known positions $\mathbf{x}_i = [x_i, y_i, z_i]^T, i = 1, 2, \cdots, n$, float in the water. We assume that $z_i = 0, i = 1, 2, \cdots, n$. The anchors broadcast signals to provide localization and clock synchronization services for underwater agents. All the anchors synchronized with the universal time clock (UTC) by GPS. The agents have their own clocks and are not synchronized. The relationship between the anchors and the ith agent is $t_i = a_i t + b_i$, where t_i is the i^{th} agent's clock, and t is the anchors' clock, a_i and b_i are the clock skew and the clock offset of the ith agent, respectively. The agents perform synchronization and localization by measuring the ToAs of the broadcast packets of anchors. Assume that the agents equip with the pressure sensors. Hence its depth coordinate can be obtained by the pressure sensor. Now we focus on a single agent $\mathbf{x} = [x, y, z]^T$ whose clock skew and clock offset are modeled as a and b, respectively. The parameter z is known via pressure sensors. The parameters which we need to estimate are the sound propagation speed c, the clock skew a, the clock offset b, the coordinates of the agent.

Assume that all the anchors transmit the k^{th} signals with the UTC time $t_{0,k}$, which satisfies $t_{0,k} = t_{0,k-1} + T, k = 1, 2, \cdots$, where T is the time interval between two consecutive signal transmission. The clock skew, the clock offset, the sound propagation speed, the positions of anchors and agents remain unchanged during two consecutive time intervals. The measured arrival times of the k^{th} transmitted signals from the anchor i to the agent $\mathbf{x}$ and to the anchor j are denoted by $\tilde{t}_{i,k}$ and $\tilde{t}_{ij,k}$ respectively, which satisfy the following equations

$$\tilde{t}_{ij,k} = t_{0,k} + \tau_{ij,k} + \varepsilon_{ij,k} \tag{1}$$

$$\tilde{t}_{i,k} = a t_{0,k} + b + a\tau_{i,k} + \varepsilon_{i,k} \tag{2}$$

$$i = 1, 2, \cdots, n; j \neq i; k = 1, 2, \cdots$$

where $\tau_{i,k}$ is the propagation time between the anchor i and the agent $\mathbf{x}$, and $\tau_{ij,k}$ is the propagation time between the anchor i and j . The time measurement error $\varepsilon_{i,k}$, and $\varepsilon_{ij,k}$, are independent of each other and zero-mean Gaussian random variables. Expressing the time delay with the ratio of the distance and the sound propagation speed, (1) and (2) can be rewritten in a compact form

$$\tilde{\mathbf{t}}_{a,k} = \mathbf{h}_a c^{-1} + \boldsymbol{\varepsilon}_{a,k} \tag{3}$$

$$\tilde{\mathbf{t}}_k - (a t_{0,k} + b)\mathbf{1} = \frac{a}{c}\mathbf{d}(\mathbf{x}) + \boldsymbol{\varepsilon}_k \tag{4}$$

where

$$\begin{aligned}
&\tilde{\mathbf{t}}_{a,k} = [\tilde{\mathbf{t}}_{a1,k}, \cdots, \tilde{\mathbf{t}}_{an,k}]^T \\
&\tilde{\mathbf{t}}_{ai,k} = [\tilde{t}_{i1,k}, \cdots, \tilde{t}_{i(i-1),k}, \tilde{t}_{i(i+1),k}, \cdots, \tilde{t}_{in,k}] - t_{0,k}\mathbf{1}^T \\
&\mathbf{h}_a = [\mathbf{h}_{a1}, \cdots, \mathbf{h}_{an}]^T \\
&\mathbf{h}_{ai} = [d_{i1}, \cdots, d_{i(i-1)}, d_{i(i+1)}, \cdots, d_{in}] \\
&d_{ij}(\mathbf{x}) = \|\mathbf{x}_i - \mathbf{x}_j\| \\
&\tilde{\mathbf{t}}_k = [\tilde{t}_{1,k}, \tilde{t}_{2,k}, \cdots, \tilde{t}_{n,k}]^T \\
&\mathbf{d}(\mathbf{x}) = [d_1(\mathbf{x}), \cdots, d_n(\mathbf{x})]^T \\
&d_i(\mathbf{x}) = \|\mathbf{x} - \mathbf{x}_i\| \\
&\boldsymbol{\varepsilon}_{a,k} = [\varepsilon_{12,k}, \cdots, \varepsilon_{1n,k}, \varepsilon_{21,k}, \varepsilon_{23,k}, \cdots, \varepsilon_{2n,k}, \cdots, \varepsilon_{n(n-1),k}]^T \\
&\boldsymbol{\varepsilon}_k = [\varepsilon_{1,k}, \varepsilon_{2,k}, \cdots, \varepsilon_{n,k}]^T
\end{aligned}$$

III. The Algorithm

In order to reduce the complexity, we first estimate the sound propagation speed c and the clock skew a of agent by the utilization of measurement information between anchors and the information of two consecutive signal transmission periods. Sequentially the weighted least squares (WLS) algorithm is utilized to estimate the remaining parameters.The details are as follows.

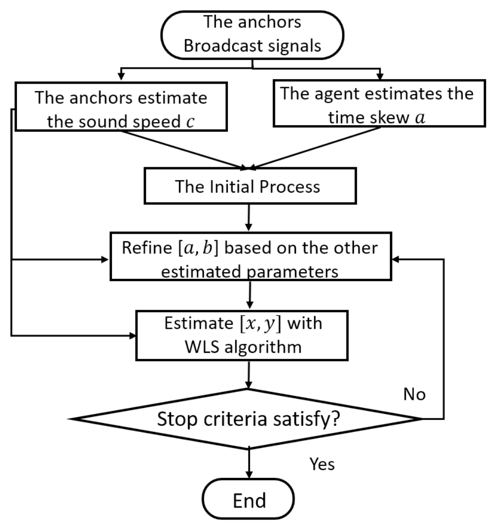

Fig. 1: Flow diagram

A. The Estimation of sound propagation speed and Clock Skew

The signals received by the anchors can be utilized to estimate the sound propagation speed. The estimated of the sound propagation speed can be broadcast to the agents in the the next time interval. We transform (3) into the following form

$$\mathbf{H}_a = \tilde{\mathbf{t}}_{a,k} c - c\boldsymbol{\varepsilon}_{a,k} \tag{5}$$

As a result, the sound propagation speed can be estimated as

$$\hat{c} = (\tilde{\mathbf{t}}_{a,k}^T \tilde{\mathbf{t}}_{a,k})^{-1} \tilde{\mathbf{t}}_{a,k}^T \mathbf{H}_a \tag{6}$$

All the signals received by the agent during the two consecutive time intervals can be combined to estimate a. According to (2), it is easy to deduce the following relationship

$$\tilde{t}_{i,k+1} - \tilde{t}_{i,k} = a(t_{0,k+1} - t_{0,k}) + (\varepsilon_{i,k+1} - \varepsilon_{i,k}) \quad i = 1, 2, \cdots, n \tag{7}$$

It can be rewritten in a compact form as

$$\boldsymbol{\beta} = aT\mathbf{1} + \boldsymbol{\varepsilon}_{k+1} - \boldsymbol{\varepsilon}_k \tag{8}$$

where $\mathbf{1} = [1, 1, \cdots, 1]^T \in \mathbb{R}^{n\times 1}$, $\boldsymbol{\beta} = [\tilde{t}_{1,k+1} - \tilde{t}_{1,k}, \tilde{t}_{2,k+1} - \tilde{t}_{2,k}, \cdots, \tilde{t}_{n,k+1} - \tilde{t}_{n,k}]^T$. Thus, the clock skew a can be obtained through the LS method

$$\hat{a} = \frac{\mathbf{1}^T\boldsymbol{\beta}}{nT} \tag{9}$$

B. The Alternative Algorithm

The alternative algorithm is introduced in this section. We partition the parameters into two groups: $[a, b]^T$ and $[x, y]^T$. The initial process is performed to obtain the initial estimate of both groups. After that, during the r^{th} iteration, we treat one of the two group as the unknown parameters and estimate them based on the other group's previous estimates, and repeat with respect to the other group until the stop condition is satisfied.

1) The Initial Process: In this part we estimate $[x, y, b]^T$ with WLS algorithm based on the estimated the clock skew $\hat{a}$ and the sound propagation speed $\hat{c}$. However, (2) is a nonlinear equation about the parameters which need to be estimated. Therefore, the first step is to linearize it. We square both sides of it and make differentiation with respective to the last equation of (2) and rearrange this results. Finally, we get the following linear equation:

$$\boldsymbol{\eta} = \mathbf{A}[x, y, b]^T + \boldsymbol{\omega} \tag{10}$$

where $\mathbf{A} = [\mathbf{a}_1, \cdots, \mathbf{a}_{n-1}]^T$, $\boldsymbol{\eta} = [\eta_1, \cdots, \eta_{n-1}]^T$ and the noise vector $\boldsymbol{\omega} = [\omega_1, \cdots, \omega_{n-1}]^T$. Their elements are shown as follows:

$$\mathbf{a}_i = [2(x_n - x_i), 2(y_n - y_i), \frac{2\hat{c}^2}{(\hat{a})^2}(\tilde{t}_{i,k} - \tilde{t}_{n,k})]^T$$

$$\eta_i = (\frac{\hat{c}}{\hat{a}}\tilde{t}_{i,k} - \hat{c}t_{0,k})^2 - (\frac{\hat{c}}{\hat{a}}\tilde{t}_{n,k} - \hat{c}t_{0,k})^2 + \|\mathbf{x}_n\|^2 - \|\mathbf{x}_i\|^2$$

$$\omega_i = 2\frac{\hat{c}}{\hat{a}}d_i(\mathbf{x})(\varepsilon_{i,k} - \varepsilon_{n,k}) + \frac{\hat{c}^2}{(\hat{a})^2}(\varepsilon_{i,k}^2 - \varepsilon_{n,k}^2)$$

The WLS algorithm is used here to solve (10). Denote the weight matrix as $\mathbf{W} \in \mathbb{R}^{(n-1)\times(n-1)}$, and initialize it with $n-1$ dimensions unit matrix. After an estimate of $[x, y]$ is obtained, we update $\mathbf{W}$ with $\mathbf{C}^{-1}$, where $\mathbf{C}$ is the convariance matrix and can be calculated through the following equation:

$$[\mathbf{C}]_{ij} = \begin{cases} \frac{4\hat{c}^2\sigma^2}{\hat{a}^2}(\frac{\hat{c}^2\sigma^2}{\hat{a}^2} + \frac{3\hat{c}\sigma^2}{\hat{a}}d_i(\hat{\mathbf{x}}) + 2(d_i(\hat{\mathbf{x}}))^2) & \text{if } i = j \\ \frac{2\hat{c}^2\sigma^2}{\hat{a}^2}(2d_i(\hat{\mathbf{x}})d_j(\hat{\mathbf{x}}) + \frac{4\hat{c}^2\sigma^2}{\hat{a}^2}) & \text{if } i \neq j \end{cases} \tag{11}$$

The convariance matrix is dependent on the distances between the anchors and the agent. Therefore, every time we get the estimate of x, y, the weigh matrix is updated. The estimators of x and y are refined until the results converge. By applying the WLS to (10), we can obtain the estimator

$$[\hat{x}, \hat{y}, \hat{b}]^T = (\mathbf{A}^T\mathbf{W}^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}^{-1}\boldsymbol{\eta} \tag{12}$$

2) The Alternative Iteration: In order to refine the estimation accuracy of them, based on the value of $\hat{x}, \hat{y}$, we estimate $[a, b]^T$ again with the LS algorithm according to (4). The estimate of $[x, y]^T$ is refined with the WLS based on $[\hat{a}, \hat{b}]^T$. The alternative iteration continues until the stop condition is satisfied. During the r^{th} iteration, substitute $\hat{x}^{(r-1)}, \hat{y}^{(r-1)}$ into (4) and rearrange it, we can deduce the linear equation of a, b, which can be stated as

$$\tilde{\mathbf{t}}_k = [t_{0,k}\mathbf{1} + \frac{1}{\hat{c}}\mathbf{d}(\hat{\mathbf{x}}^{(r)}), \mathbf{1}][a, b]^T + \boldsymbol{\varepsilon}_k \tag{13}$$

Let $\mathbf{Q}(\hat{\mathbf{x}}) \triangleq [t_{0,k}\mathbf{1} + \frac{1}{\hat{c}}\mathbf{d}(\hat{\mathbf{x}}), \mathbf{1}]$. The estimator of $[a, b]^T$ can be expressed as

$$[\hat{a}^{(r)}, \hat{b}^{(r)}]^T = (\mathbf{Q}^T(\hat{\mathbf{x}}^{(r-1)})\mathbf{Q}(\hat{\mathbf{x}}^{(r-1)}))^{-1}\mathbf{Q}^T(\hat{\mathbf{x}}^{(r-1)})\tilde{\mathbf{t}}_k \tag{14}$$

Substitute $[\hat{a}^{(r)}, \hat{b}^{(r)}]^T$ into (10) and rearrange it. We have

$$\boldsymbol{\mu} = \mathbf{B}[x, y]^T + \boldsymbol{\omega} \tag{15}$$

where $\mathbf{B} = [\mathbf{b}_1, \cdots, \mathbf{b}_{n-1}]^T$, $\boldsymbol{\mu} = [\mu_1, \cdots, \mu_{n-1}]^T$. $\mathbf{b}_i = [2(x_n - x_i), 2(y_n - y_i)]^T$ and $\mu_i = (\frac{\hat{c}}{\hat{a}^{(r)}}\tilde{t}_{i,k} - \hat{c}t_{0,k})^2 - (\frac{\hat{c}}{\hat{a}^{(r)}}\tilde{t}_{n,k} - \hat{c}t_{0,k})^2 + \|\mathbf{x}_n\|^2 - \|\mathbf{x}_i\|^2 - \frac{2\hat{c}^2}{(\hat{a}^{(r)})^2}(\tilde{t}_{i,k} - \tilde{t}_{n,k})\hat{b}^{(r)}$. The WLS estimator is given by

$$[\hat{x}^{(r)}, \hat{y}^{(r)}]^T = (\mathbf{B}^T\mathbf{W}^{-1}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{W}^{-1}\boldsymbol{\mu} \tag{16}$$

where $W = C^{-1}$. We decide whether to stop according to the difference between $[\hat{a}^{(r)}, \hat{b}^{(r)}, \hat{x}^{(r)}, \hat{y}^{(r)}]$ and $[\hat{a}^{(r-1)}, \hat{b}^{(r-1)}, \hat{x}^{(r-1)}, \hat{y}^{(r-1)}]$.

IV. Cramér-Rao Bound

The CRB states that the variance of any unbiased estimator is at least as high as the inverse of the Fisher information. In order to evaluate the performance of of our proposed estimators, we compare them with the corresponding CRBs. We first show that our estimator are unbiased estimators, then calculate the CRB of the sound propagation speed c, the clock skew a, the clock offset b, and the x , y coordinates of the agent. As the estimation of the sound propagation speed c is based on the informations obtained by anchors, while the estimation of the other parameters $a, b, x, y,$, is based on the informations obtained by the agent, we analyze the CRB of them separately. Before that the unbiased characteristic of our estimator is shown first. According to the estimator (6) of sound propagation speed c, the expectation of $\hat{c}$ can be calculated as

$$\begin{aligned} E[\hat{c}] &= E[(\tilde{\mathbf{t}}_{a,k}^T\tilde{\mathbf{t}}_{a,k})^{-1}\tilde{\mathbf{t}}_{a,k}^T\mathbf{H}_a] \\ &= c - c[(\tilde{\mathbf{t}}_{a,k}^T\tilde{\mathbf{t}}_{a,k})^{-1}\tilde{\mathbf{t}}_{a,k}^T]E[\boldsymbol{\varepsilon}_{a,k}] = c \end{aligned} \tag{17}$$

We have estimated a and b twice: the first time we estimated a alone, and the second time we estimated them together. Therefore both estimators are needed to be check. First, Substitute (8) into (9) and calculate the expectation. We have

$$
\begin{aligned}
E[\hat{a}] &= E[\frac{\mathbf{1}^T\boldsymbol{\beta}}{nT}] \\
&= a + \frac{1}{nT}\sum_{i=1}^{n} E(\varepsilon_{i,k+1} - \varepsilon_{i,k}) = a
\end{aligned}
$$

Combining (13) and (14), and taking expectation with respect to $[\hat{a}, \hat{b}]^T$, we have

$$
\begin{aligned}
E\{[a,b]^T\} &= E\{(\mathbf{Q}^T(\hat{\mathbf{x}})\mathbf{Q}(\hat{\mathbf{x}}))^{-1}\mathbf{Q}^T(\hat{\mathbf{x}})(\mathbf{Q}(\hat{\mathbf{x}})[a,b]^T + \boldsymbol{\varepsilon}_k)\} \\
&= [a,b]^T
\end{aligned}
$$

Similarly, by substituting (10) into (12), we can calculate the expectation of $[x,y,b]^T$ as

$$
\begin{aligned}
E\{[\hat{x},\hat{y},\hat{b}]^T\} &= E\{(\mathbf{A}^T\mathbf{W}^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}^{-1}(A[x,y,b]^T + \boldsymbol{\omega})\} \\
&= [x,y,b]^T + (\mathbf{A}^T\mathbf{W}^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}^{-1}E(\boldsymbol{\omega}) \\
&= [x,y,b]^T
\end{aligned}
$$

The estimator of $[x,y]^T$ is also an unbiased estimator, as we show it as follows:

$$
\begin{aligned}
E\{[\hat{x}^{(r)},\hat{y}^{(r)}] &= E\{(\mathbf{B}^T\mathbf{W}^{-1}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{W}^{-1}(\mathbf{B}[x,y]^T + \boldsymbol{\omega})\}\} \\
&= [x,y]^T + E\{(\mathbf{B}^T\mathbf{W}^{-1}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{W}^{-1}\boldsymbol{\omega}\} \\
&= [x,y]^T
\end{aligned}
$$

Therefore, the estimator(6), (9), (12), (14) and (16) are unbiased estimators. Now we analyze the unbiased CRB of all the parameters. According to (1), we denote $\tilde{T}_{a,k} = \tilde{\mathbf{t}}_{a,k} + t_0^{(k)}\mathbf{1}^T \in \mathbb{R}^{n(n-1)\times 1}$, then the time measurement of the anchors can be modeled as

$$
\tilde{T}_{a,k} \sim \mathcal{N}(F_{a,k}(c), \sigma^2) \tag{18}
$$

where $F_{a,k}(c) = t_{0,k}\mathbf{1}^T + \mathbf{H}_a c^{-1} \in \mathbb{R}^{n(n-1)\times 1}$. The probability density function (PDF) of $\tilde{T}_{a,k}$ is

$$
\begin{aligned}
&p(\tilde{\mathbf{t}}_a^{(k)}; c) = \\
&\frac{1}{(2\pi\sigma^2)^{n(n-1)/2}}\exp\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}(\tilde{t}_{ij,k} - t_{0,k} - d_{ij,k}/c)^2\}
\end{aligned}
$$

Then the CRB of c can be expressed as

$$
\begin{aligned}
\text{CRB}(\hat{c}) &= -E^{-1}[\frac{\partial^2 \ln p(\tilde{T}_{a,k}; c)}{\partial c^2}] \\
&= \frac{\sigma^2 c^4}{\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n} d_{ij}^2}
\end{aligned} \tag{19}
$$

Similarly, we can described the time measurement of the agent in the following form

$$
\tilde{\mathbf{t}}_k \sim \mathcal{N}(F_k(\boldsymbol{\theta}), \sigma^2\mathbf{I}_n) \tag{20}
$$

where

$$
\begin{aligned}
F^{(k)}(\boldsymbol{\theta}) &= [f_{1,k}(\boldsymbol{\theta}),, \cdots, f_{n,k}(\boldsymbol{\theta})]^T \in \mathbb{R}^{n\times 1} \\
f_i^{(k)} &= at_{0,k} + b + a\tau_i \\
\boldsymbol{\theta} &= [\theta_1, \theta_2, \theta_3, \theta_4]^T \triangleq [a,b,x,y]^T
\end{aligned}
$$

The corresponding Fisher information matrix can be stated as

$$
[\mathbf{M}(\boldsymbol{\theta})]_{ij} = \frac{1}{\sigma^2}[\frac{\partial F_k(\boldsymbol{\theta})}{\partial \theta_i}]^T[\frac{\partial F_k(\boldsymbol{\theta})}{\partial \theta_j}] \tag{21}
$$

where

$$
\begin{aligned}
\frac{\partial F_k(\boldsymbol{\theta})}{\partial \theta_i} &= [\frac{\partial f_{1,k}}{\partial \theta_i}, \cdots, \frac{\partial f_{n,k}}{\partial \theta_i}]^T \\
\frac{\partial f_{i,k}}{\partial \theta_1} &= t_{0,k} + \frac{d_i(\mathbf{x})}{c} \\
\frac{\partial f_{i,k}}{\partial \theta_2} &= 1 \\
\frac{\partial f_{i,k}}{\partial \theta_4} &= \frac{a(x - x_i)}{cd_i(\mathbf{x})} \\
\frac{\partial f_{i,k}}{\partial \theta_5} &= \frac{a(y - y_i)}{cd_i(\mathbf{x})} \\
i &= 1, 2, \cdots, n
\end{aligned}
$$

and the CRB of $\boldsymbol{\theta}$ can be expressed as

$$
\text{Var}(\hat{\theta}_i) \geq= [\mathbf{M}^{-1}(\boldsymbol{\theta})]_{ii} \tag{22}
$$

V. Simulation

The performance of our algorithm is evaluated through numerical simulations. There are 16 anchors floating on the water. They broadcast signals every $T = 0.1s$, and measure the arrival times of the other anchors' signals to estimate the sound propagation speed. The agent receives all the signals, measures their arrival times, and estimates the other parameters.

Fig. 2 shows the square root of CRB and the root mean square error (RMSE) of the sound propagation speed. As it is shown the RMSE is almost equal to the CRB along the noise axis. Therefore, our sound propagation speed estimator performs well.

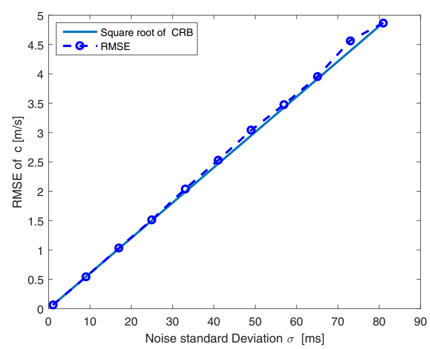

Fig. 2: The RMSE and $\sqrt{\text{CRB}}$ of sound propagation speed c. The maximum of noise standard Deviation is 80ms

The RMSE and the $\sqrt{\text{CRB}}$ of the parameters a, b of agent are shown in Fig. 3 and 4. As they show, the RMSEs are very close to $\sqrt{\text{CRB}}$ when the noise standard deviation is small enough. Although the difference between the RMSE and $\sqrt{\text{CRB}}$continues increasing with the increasing of noise standard deviation, the differences are small.

It is similar to the estimators of a and b, the RMSE of the agent's location is almost equal to the corresponding CRB when the noise standard deviation is small enough as shown in Fig. 5. While the difference also keep increasing with the increasing of noise standard deviation.

VI. Conclusion

In this paper, we propose an underwater acoustic localization algorithm with the consideration of uncertainties in sound propagation

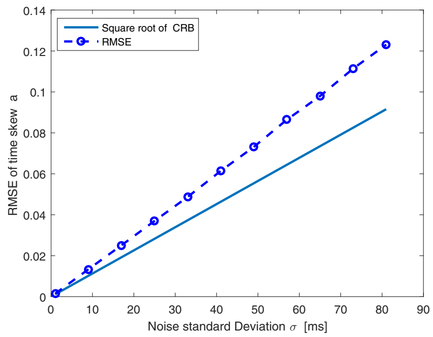

Fig. 3: The RMSE and $\sqrt{\text{CRB}}$ of the agent's clock skew a. The maximum of noise standard deviation is 80ms

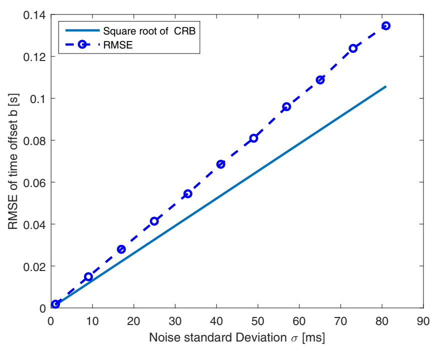

Fig. 4: The RMSE and $\sqrt{\text{CRB}}$ of the agent's clock offset b. The maximum of noise standard deviation is 80ms

speed and time synchronization. The anchors first broadcast signals, then estimate the sound propagation speed via the ToAs of the signals from the other anchors, and send the estimated sound propagation speed to the underwater agent in the second time interval. The agent measures the ToAs of the signals broadcast by the anchors, and combine the ToAs measured in two consecutive intervals to estimate the clock skews. After that the estimation of the agent positions and the clock offsets is performed with the WLS algorithm. Finally, we

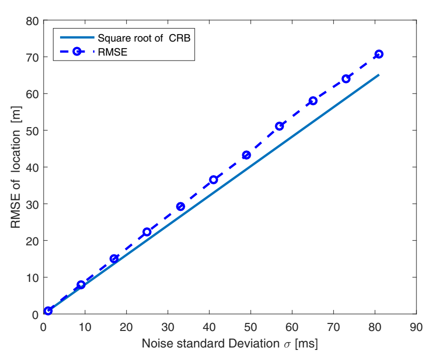

Fig. 5: The RMSE and $\sqrt{\text{CRB}}$. of the agent's location $[x, y, z]^T$. The maximum of noise standard deviation is 80ms

perform the alternative iteration process to refine the accuracy of a, b, x, y.

Our method does not require the agents to transmit any signal, which will prolong the life of the agents. The proposed estimator are unbiased. The estimation of all parameters: the sound propagation speed, the clock skew, the clock offset and the agent's position only takes two consecutive time intervals. Therefore the localization period of our method is small. The simulations show that the RMSEs of the sound propagation speed is almost equal to the corresponding $\sqrt{\text{CRB}}$. While, the differences between the RMSE and the corresponding $\sqrt{\text{CRB}}$ of the other estimators remain increasing along the noise standard deviation axis. But, the differences are still relative small. Especially when the noise standard deviation is small, such as $\sigma < 10ms$, the RMSE is get so close to the corresponding $\sqrt{\text{CRB}}$.

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