# Belief Propagation Based Multi-AUV Cooperative Localization in Anchor-free Environments

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***Abstract*—The localization of autonomous underwater vehicles (AUVs) has been a difficult yet fundamental issue in many applications. The traditional way to localize an AUV is based on dead-reckoning (DR) using the measurements from inertial measurement units (IMUs). However, the accuracy of DR cannot be guaranteed for a long period. With the development of underwater communications and ranging, recent AUVs can localize themselves by sharing position information with anchors (whose positions are known). Occasionally, we have to localize an AUV in anchor-free environments. Without reference positions, localization could be challenging. Recently, multiple-AUV (multi-AUV) simultaneous navigation is becoming a prevalent trend and cooperative localization becomes a new way to improve the localization accuracy. In this paper, aiming at anchor-free scenarios, we propose a novel cooperative localization algorithm using belief propagation (BP), called belief propagation based dead-reckoning (BPDR). Meanwhile, to reduce the communication overhead among AUVs, intermittent BPDR (IBPDR) algorithm is designed. The simulations show satisfactory performance in localization by both BPDR and IBPDR.**

## I. Introduction

Ocean exploitation is becoming more and more important for human beings. Autonomous underwater vehicles (AUVs) are becoming indispensable due to their high flexibility and long-range navigation ability. When carrying out a task, an AUV's location information is fundamental and determines the ability of completing the mission. However, AUV localization has always been a challenge in underwater applications because of lack of Global Positioning System (GPS) signals, limitations of acoustic communications, unpredictable underwater environments and etc. The methods in AUV localization have been surveyed in [1]. So far, the methods can be roughly divided into three types [2]: i) inertial localization; ii) acoustic localization; and iii) geophysical localization. The inertial localization uses measurements obtained from inertial measurement units (IMUs) to estimate the position by dead-reckoning (DR). Nevertheless, for a long time, the estimation error can be several sea miles per hour [3]. Long baseline (LBL), short baseline (SBL) and ultrashort baseline (USBL) are typical methods of acoustic localization. Although the estimation accuracy could be high, the cost of system maintenance and limited service region restrict their applications. Geophysical localization is a relatively new branch of localization techniques, using physical characteristics of the surroundings to localize AUVs.

Recently, multiple AUVs simultaneously carrying out a task becomes popular with the development of underwater communications and navigation. In addition, multiple-AUV (multi-AUV) cooperative navigation can provide a better localization performance [4]. Leader-follower is a popular pattern of cooperation [5], where the leader AUV is always armed with high-precision sensors or frequently surfaces to obtain GPS signals. Cooperation with communication and navigation aids (CNAs) is similar to leader-follower. It is applied in [6]–[8], where CNAs are always surface vehicles with the ability of receiving GPS signals. As for localization algorithms, extended Kalman filter (EKF) is a popular one [8]–[10]. Belief propagation (BP) is also a well-known range-based cooperative localization algorithm which is widely used in wireless networks (statical and mobile) [11]–[13]. Its naturally distributed structure is suitable for multi-AUV cooperative localization. Moreover, BP can provide not only the location estimates but also the uncertainties of the location estimates. Up to now, most research works involve anchors or devices with known positions. However, there are many cases where no anchors are available. In such situation, with no absolute reference, localization is more difficult.

In this paper, we propose cooperative localization algorithms for multiple AUVs with underwater communication and ranging capabilities aiming at anchor-free environments. Making use of range measurements and DR, we first propose a BP-type algorithm, called belief propagation based dead-reckoning (BPDR). Moreover, to reduce the communication cost of cooperation among AUVs, we further design an algorithm called intermittend BPDR (IBPDR). The simulation results show good performance of the proposed algorithms. The rest of this paper is arranged as follows. The system model is described in Section II. In Section III, the proposed cooperative

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localization algorithms are represented. The simulation results are delivered in Section IV.

## II. System Model

In this paper, a team of four AUVs is considered in an anchor-free environment with low possibility or even no possibility of emerging from the sea to gain GPS signals. Such environment is very typical. For example, the deep sea where frequent surfacing is not feasible, or the polar regions covered with ice. Although the underwater environment is a 3D space, every AUV carries a depth sensor, by which the AUV's depth is measured. Without loss of generality, we can simplify the position model to a 2D case with coordinates $\boldsymbol{x}_i^k = [\alpha_i^k, \beta_i^k]^T$. The superscript and subscript indicate the time index and the AUV index, respectively. In this paper, we omit the superscript for simplicity unless it is necessary. Fig. 1 shows the scenario.

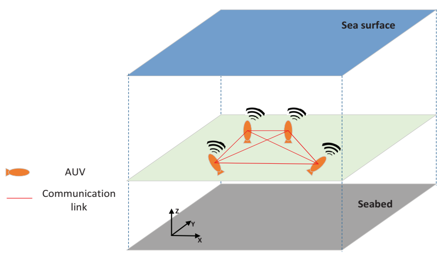


Fig. 1: System Model

Except for the depth sensor, we assume that every AUV is equipped with navigation sensors, such as IMUs and Doppler velocity log (DVL), and acoustic modems with ranging capability to measure the distances between AUVs. The IMUs and DVL are the sources of measurements which are mainly used in DR to deduce the AUV's position. Furthermore, in this paper, we assume that the quality of acoustic communication is always fine.

The acoustic modems measure the relative distance between two AUVs, which can be modeled as

$$z_{ij} = d_{ij} + n_{ij}, \tag{1}$$

$$= \|\boldsymbol{x}_i - \boldsymbol{x}_j\| + n_{ij}, \tag{2}$$

where $d_{ij}$ is the real distance between the $i$th and $j$th AUVs, $\boldsymbol{x}_i = [\alpha_i, \beta_i]^T$ is the position of the $i$th AUV. The measured distance is corrupted by additive zero mean Gaussian noise $n_{ij}$ with identical variance $\sigma_{ij}^2 = \sigma^2$ for all AUVs. Nevertheless, the Gaussian measurement model may not be enough for practical applications. In localization problems, non-Gaussian situations widely exist [14]. Fortunately, the algorithms we propose in the next section are particle-based. Thus, the algorithms can be easily extended to non-Gaussian measurement models.

## III. The Algorithms of Cooperative Localization

In this section, the details of the proposed cooperative localization algorithms are described. We first introduce the core techniques used in the algorithms. Then the proposed cooperative methods are introduced.

### A. Dead-Reckoning

DR is widely used in many fields. As long as a vehicle is armed with IMUs, the problem can be solved by DR. Since GPS signals are not available underwater, DR becomes significantly important in AUV navigation. Using the navigation orientation and the velocity obtained from sensors, we can calculate the position $\boldsymbol{x}^{k+1} = [\alpha^{k+1}, \beta^{k+1}]^T$ (we omit the subscript in this subsection, as DR is the same for all the AUVs) at the time step $k+1$ with the known previous position $\boldsymbol{x}^k = [\alpha^k, \beta^k]^T$ using DR

$$\alpha^{k+1} = \alpha^k + v^k t sin\theta^k, \tag{3}$$

$$\beta^{k+1} = \beta^k + v^k t cos\theta^k, \tag{4}$$

where $v^k$ is the velocity of the vehicle at the time step $k$, $t$ is the time duration between two time steps, and the vehicle's heading angle is described by $\theta^k$.

One of DR's important advantages is that it needs measurements only from internal sensors. No communications with other devices are required. This strength also makes the vehicle stay mute which is necessary in some circumstances. However, the weakness of DR is also obvious that the accuracy is guaranteed only for a short time period and the position estimation error grows without bound as time goes on [2].

The algorithms we propose in Section III-C and III-D make use of DR and cooperations among AUVs, devoting to slow down its divergence rate and guarantee the estimation accuracy for a long time period.

### B. Belief Propagation

In this subsection, we describe the basic mathematical principles of the belief propagation and how it is applied in localization.

Graphs can be used to solve inference problems, in which each node is associated with a random variable and represents a probability distribution. BP is a message-passing algorithm based on graphs for distributed inference. To illustrate the principle of BP, we first define a set of nodes $\mathcal{V}$ associated with a variable set $\mathcal{X}$ and a set of edges $\mathcal{E}$ in which every edge connects two nodes $\{i, j \in \mathcal{V}\}$.

Based on the graphical model, the joint posterior distribution $p(\boldsymbol{X}|\boldsymbol{Y})$ can be derived [15]

$$p(\boldsymbol{X}|\boldsymbol{Y}) = \prod_{(i,j)\in\mathcal{E}} \psi_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j) \prod_{i\in\mathcal{V}} \psi_i(\boldsymbol{x}_i), \tag{5}$$

where $\boldsymbol{Y}$ indicates the observations of the random variables $\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_n]^T$, $\boldsymbol{x}_i \in \mathcal{X}$, and $n$ is the number of nodes in the network. The functions $\psi_{ij}$ and $\psi_i$ are potential functions in graphical models, describing the pairwise relations between nodes and local information of a single node, respectively. For localization, the position of an AUV can be viewed as the state of a variable $\boldsymbol{x}_i$, the function $\psi_i(\boldsymbol{x}_i)$ denotes the prior position distribution $p_i(\boldsymbol{x}_i)$ and the function $\psi_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j)$ is described by the measurement likelihood function $p(d_{ij}|\boldsymbol{x}_i, \boldsymbol{x}_j)$ [16].

For a certain variable $\boldsymbol{x}_i$, our goal is to calculate its marginal posterior distribution $p(\boldsymbol{x}_i|\boldsymbol{Y})$, and use minimum mean squared error (MMSE) criteria (or maximum posterior (MAP) criteria) to obtain its estimated state (position) $\hat{\boldsymbol{x}}_i = \int \boldsymbol{x}_i p(\boldsymbol{x}_i|\boldsymbol{Y})\, d\boldsymbol{x}_i$. The conventional way of obtaining the marginal posterior distribution of variable $\boldsymbol{x}_i$ is given below

$$p(\boldsymbol{x}_i|\boldsymbol{Y}) = \int p(\boldsymbol{X}|\boldsymbol{Y})\, d \sim \{\boldsymbol{X}\}\,, \qquad (6)$$

where $\sim \{\boldsymbol{X}\}$ denotes all the variables in $\boldsymbol{X}$ except $\boldsymbol{x}_i$. Now a problem is arisen that the straightforward computation of (6) is usually impractical.

BP is an iterative message-passing algorithm, also known as the sum-product algorithm, designed to calculate the marginal distribution $p(\boldsymbol{x}_i|\boldsymbol{Y})$ [15]. The process of BP is simple. In every iteration, every node computes the messages and sends them to its neighbors, then calculates its own "belief" (an approximation of the marginal posterior distribution of the associated variable $\boldsymbol{x}_i$) with received messages.

Let us define the neighbors of node $i$ as $\boldsymbol{N}(i)$. The message sent from node $i$ to one of its neighbors $j \in \boldsymbol{N}(i)$ can be obtained as [15]

$$m_{i\to j}(\boldsymbol{x}_j) \propto \int \psi_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j)\psi_i(\boldsymbol{x}_i) \prod_{k\in \boldsymbol{N}(i)\backslash j} m_{k\to i}(\boldsymbol{x}_i) d\boldsymbol{x}_i. \qquad (7)$$

The message is defined as a combination of pairwise relation between two nodes, the local information of the node $i$ and the incoming messages from other nodes. After receiving all the messages from neighbors, the "belief" of each node can be calculated by

$$b_i(\boldsymbol{x}_i) \propto \psi_i(\boldsymbol{x}_i) \prod_{k\in \boldsymbol{N}(i)} m_{k\to i}(\boldsymbol{x}_i). \qquad (8)$$

If the graph is tree-structured, the "belief" of node $i$ will eventually converge to $p(\boldsymbol{x}_i|\boldsymbol{Y})$.

From (7) and (8), we can implement BP to get the approximate marginal distribution. However, the closed-form solutions of (7) and (8) are available only under Gaussian assumption (continuous version) [12]. The Gaussian uncertainty model is problematical in practice, as non-Gaussian situations widely exist. To ensure the usability of BP in AUV localization, we apply an extended version of BP, based on particle methods, called nonparametric belief propagation (NBP), which is suitable for localization. The main idea of NBP is that using particles to represent the position distributions of nodes. In this paper, our implementation of BP is based on NBP. More details about NBP and its applications in localization are shown in [14].

### C. Cooperative Navigation Using BPDR

In anchor-free environments, without any absolute position information, DR is the only solution for single-AUV localization. Nevertheless, [4] has proved that cooperative methods for multiple AUVs can improve the position estimation accuracy. Thus, we propose a cooperative localization algorithm using both DR and BP.

The most common way of localization without anchor is that using DR to estimate position when the AUV is underwater and frequent surfacing to fix DR's deviation by GPS. In reality, frequent surfacing is not a good option. Because it is energy-consuming or impossible in ice-crusted regions. The main idea of our algorithms is that using the BP localization method to replace the GPS-fix (surfacing) process. Thus, we can slow down the divergence rate of DR, and avoid surfacing energy consumption.

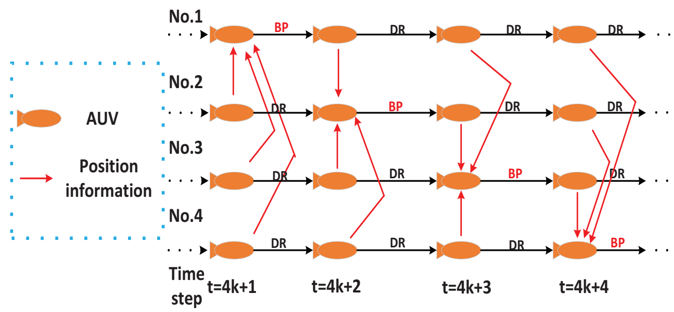

Fig. 2: Procedure of BPDR

To illustrate the procedure of BPDR, shown in Fig. 2, we label our four AUVs as $No.1$ to $No.4$, and define three kinds of time set $\mathcal{T}^i_{DR}$, $\mathcal{T}^i_{BP}$ and $\mathcal{T}$. The sets $\mathcal{T}^i_{DR}$ and $\mathcal{T}^i_{BP}$ contain the time steps that $No.i$ AUV will use DR or BP to estimate its position respectively. The set $\mathcal{T}$ includes all the time steps of navigation.

$$\mathcal{T}^i_{BP} = 4k + i, \quad i = 1, 2, 3, 4, \quad k \in \mathbb{N}, \qquad (9)$$

$$\mathcal{T}^i_{DR} = \mathcal{T} \backslash \mathcal{T}^i_{BP}. \qquad (10)$$

From (9) and (10) we can see that BPDR views four time steps (the number of AUVs) as a period, defined as $T_1$. Four AUVs use BP to cooperatively localize themselves in sequence periodically. In this way, the deviation of each AUV caused by DR is fixed or alleviated once in a period. For the AUVs that are just fixed by BP at the latest time step and the one before that, we view them as anchors. In this way, their position distributions can be represented only by their coordinates instead of particles.

Note that the BP we use here is an adaption of standard BP. In (8), $\psi_i(\boldsymbol{x}_i)$ can be viewed as the prior position distribution $p_i(\boldsymbol{x}^k_i)$, as stated in Section III-B. In BPDR, $p_i(\boldsymbol{x}^k_i)$ is provided by $b_i(\boldsymbol{x}^{k-4}_i)$ with four times DR calculations. Thus, $p_i(\boldsymbol{x}^k_i)$ (represented by a set of particles) could be corrupted, especially when the deviation of DR is large. Therefore, we use a Gaussian distribution, with mean $E(p_i(\boldsymbol{x}^k_i))$ ($E(\cdot)$ calculates the average value) and variance $\sigma^2$ , to approximate $p_i(\boldsymbol{x}^k_i)$.

### D. Intermittent BPDR

In wireless sensor networks (WSNs), energy conservation is important and it is more relevant underwater. In our algorithms, we use particle-based methods to implement BP in cooperative localization. Although the result can be appealing, the communication overhead of transmitting a set of particles cannot be ignored and the throughput of acoustic modems should fulfill the requirement of transmitting hundreds of particles at one time step. The works in [14] and [16] have studied some ways to reduce the communication consumption by decreasing the transmitted data volume. We reduce the communication overhead, from another perspective, by decreasing the communication frequency among AUVs. Sequentially, we propose the intermittent BPDR (IBPDR) algorithm.

In every BPDR period, each AUV localizes itself using BP in sequence. Our inspiration is to insert a silent interval (no communication happens) between two periods to decrease the information exchanging frequency. The concept of IBPDR is shown in Fig. 3.

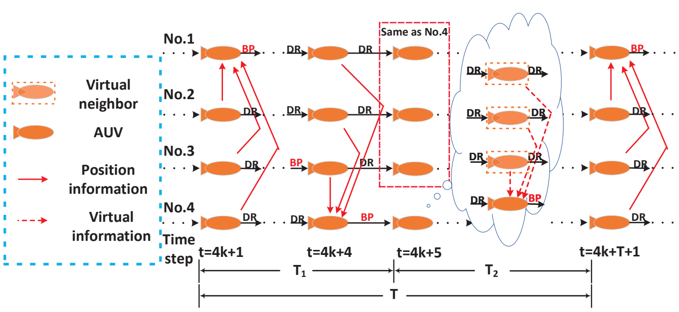


Fig. 3: Procedure of IBPDR

We first define the period of IBPDR as $T = T_1 + T_2$, in which $T_1$ is the duration of BPDR and $T_2$ indicates the silent interval. Here, we focus on the silent interval and assume that all AUVs know the arranged trajectory of every fleet member. This assumption is reasonable, since the predesigned paths of AUVs should be uploaded before the task. After the communication period (BPDR period), AUVs are aware of their neighbors' positions. On the basis of this information combined with the predesigned paths, when silent interval begins, every AUV predicts its neighbors' trajectories and treats them as the received position information from its neighbors. Then every AUV runs BP to localize itself simultaneously at every time step. Taking Fig. 3 as an example, the silent interval begins from time step $4k + 5$. $No.4$ AUV "imagines" that it receives its neighbors' position information and, accordingly, applies BP to estimate its own position. Since the neighbors are imagined, we call this kind of neighbors as "virtual neighbors".

In fact, IBPDR can be viewed as a general framework, for the use of silent interval can be various. This gives IBPDR many possibilities of further improvements.

## IV. Simulation Results

In this section, we provide simulation results about the proposed BPDR and IBPDR algorithms used in an anchor-free scenario. We consider a fleet of four AUVs navigating in a 2D plane for $T = 100$ seconds. The velocity $v$ of each AUV is 4 m/s. The measured heading angles and velocities of AUVs (obtained from IMUs and DVL respectively) are corrupted with zero mean Gaussian noise with variance $\sigma_\theta^2 = 1$ and $\sigma_v^2 = 0.04$. The simulation parameters $\sigma_\theta^2$, $\sigma_v^2$ and $v$ jointly determine the divergence rate of DR. In order to make a clear performance comparison among algorithms, the values of the parameters we choose here exaggerate the divergence rate. Thus, when $T$ reaches 100, the deviation of DR is large enough such that the performance curves of different algorithms are clearly separated.

We compare the proposed methods with DR and EKF. Note that, in both BPDR and IBPDR, the implementation of BP follows the principle of NBP for mobile networks [17] with 200 particles. The number of iterations of BP process is 1. The initial distribution of each AUV is assumed to be Gaussian $p_i(\boldsymbol{x}_i^1) = \mathcal{N}(\boldsymbol{x}_i^1, 1)$. In addition, the duration of silent interval in IBPDR is $T_2 = 10$ seconds. We implement intermittent EKF (IEKF) [18], [19] to ensure the cooperative position update rates of BPDR and EKF are equal. The state variable $\boldsymbol{x}_i^k = [\alpha_i^k, \beta_i^k, v_{\alpha,i}^k, v_{\beta,i}^k]^T$ in EKF is different from that defined in Section II, with $v_{\alpha,i}^k$ and $v_{\beta,i}^k$ denote the velocities along $x$ and $y$ axis, respectively. The state evolution equation of EKF is the same as that in [20].

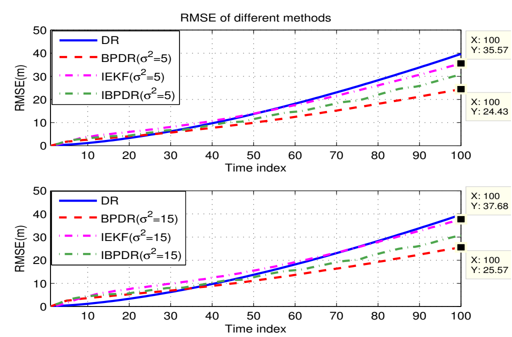


Fig. 4: The RMSE of different navigation methods

Fig. 4 shows the root-mean-square error (RMSE) of the position estimation of different algorithms with the variance of ranging error $\sigma^2 = 5$ and $\sigma^2 = 15$, respectively. The RMSE results are performed by averaging over all four AUVs with a thousand Monte Carlo simulation runs. From Fig. 4, it is obvious that the best RMSE curves are provided by BPDR in both subfigures, which also indicates a good robustness of BPDR under different measurement accuracy. When $\sigma^2$ increases from 5 to 15, BPDR suffers less accuracy loss. This demonstrates the robustness of BPDR as well. Moreover, Fig .4 reveals the nature of DR that it performs dependably only for a short period.

We can also see that the performance of IBPDR is still acceptable although there is a little accuracy loss compared with BPDR. What's important is that, the communication consumption is dramatically reduced in IBPDR. Suppose that the communication consumption of BPDR is $E$ in a duration of $T_1 + T_2$. Then in IBPDR, it reduces to $\frac{T_1 E}{T_1 + T_2}$. Usually, we would like to choose $T_2 > T_1$. This is a trade-off between estimation accuracy and communication consumption.

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