

# Robust Localization Exploiting Sparse Residuals

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## Abstract

We consider a wireless sensor network (WSN) that employs ultra-wideband impulse radio (UWB-IR) to locate a target. This can be done using the two-way ranging protocol (TWR). It can be shown that when using TWR, the localization problem can be linearized. The range measurements between the sensor nodes and the anchors are not assumed reliable, however. A way to deal with the presence of a small number of incorrect measurements is to use a robust estimator such as the least absolute shrinkage and selection operator (LASSO) or sparse residuals, instead of a least squares estimator. It is demonstrated that using a robust estimator can significantly improve the quality of the localization, provided that the number of incorrect measurements is small.

## 1 Introduction

Ultra-wideband (UWB) has been emerging as an attractive technology with interesting applications in wireless communication, localization and radar [3, 7, 12]. Due to its large bandwidth, UWB impulse radio (UWB-IR) can provide good ranging capabilities with submeter accuracy. This feature is attractive for providing localization services to wireless sensor networks (WSNs). Moreover, UWB-IR has already been included in the IEEE 802.15.4a standard for wireless personal area networks (WPANs) [1], which makes its widespread adoption a possibility.

In this work we consider a WSN that employs UWB-IR ranging to locate a target. However, we do not assume that all WSN anchor nodes report reliable range measurements: an adversary can tamper with the software of individual nodes, or exploit the nature of the wireless medium to alter the measured range, for example by using reflectors. Non-line-of-sight (NLOS) propagation could also introduce errors to the range measurement. Since it is difficult to predict and guard against every possible attack or problem, it can be more reasonable to attempt to detect and thwart any such attacks instead.

In some cases, it is reasonable to assume that out of all range measurements, only a few will be corrupted by one or more attackers. We define these as being outlying measurements, or outliers, in the sense that they are a minority in the set of all measurements. This opens the possibility of formulating the localization problem in a way that takes advantage of the expected small number of outliers. The least absolute shrinkage and selection operator (LASSO) is such a formulation and has been successfully employed in various fields such as genetics [13, 15], image processing [2], compressive sampling [4], and error correction [5] to tackle outlier presence. In some of these problems, e.g. compressive sampling, it is known a priori that the result should be a sparse vector. In other problems, such as the localization problem we examine in this work, the result is a dense vector of coordinates that yields residuals that are sparse. The popularity of the LASSO and its variants stems from the fact that it has

the parsimony property: it promotes solutions containing just a few nonzero elements, their amount being controlled by a tunable parameter  $\lambda$ .

In this paper we begin by examining the two-way ranging protocol (TWR) in Section 2 and deriving linear models for the localization problem using the method presented in [16], while taking the presence of outliers into account. We then argue in Section 3 that LASSO or sparse residuals can be applied to get a position estimate from the linear model. In Section 4 we present and discuss simulation results, whereas in Section 5 we give our conclusions.

## 2 Ranging protocol and system model

In our localization scenario, a target node is located with the help of  $M$  anchor nodes whose locations are known. Each anchor is denoted by a positive integer tag  $1, 2, \dots, M$ . The tag 0 is reserved for the target. Let  $\mathcal{I} = \{1, 2, \dots, M\}$  be the set of all anchor tags. All nodes are dispersed in a  $p$ -dimensional space  $\mathbf{x}_i = [x_{1,i}, x_{2,i}, \dots, x_{p,i}]^T$ ,  $i \in \mathcal{I}$ , where  $\mathbf{x}_i$  stacks the coordinates of the  $i$ th anchor. Vector  $\mathbf{x}$  denotes the unknown coordinates of the target node. The internal clock of each anchor and the target has the relation  $C_i(t) = t + \theta_i$ ,  $i \in \{0, 1, \dots, M\}$  with the real time  $t$ . The anchors and the target can exhibit different clock offsets, i.e.  $\theta_0 \neq \theta_1 \neq \dots \neq \theta_M$  in general.

### 2.1 TWR

The two-way ranging (TWR) protocol [1] is a simple way to provide ranging information. An anchor node  $i$  can start the protocol by sending a ranging packet. This can be any kind of packet that has the ranging bit set. The first UWB pulse of the first bit of a ranging packet is the ranging marker (RMARKER). The anchor node records a timestamp  $t_{iS0}$  when the RMARKER leaves its antenna. The target receives the RMARKER at time  $t_{0Ri}$ , carries out some internal processing and sends back a response to anchor  $i$  at time  $t_{0Si}$ . When anchor  $i$  receives the response, it records a second timestamp  $t_{iR0}$  (Fig. 1). A fusion node receives the timestamps  $t_{iS0}$ ,  $t_{0Ri}$ ,  $t_{0Si}$ ,  $t_{iR0}$  and can calculate an estimate

$$r_i = \frac{1}{2}c((t_{iR0} - t_{iS0}) - (t_{0Si} - t_{0Ri})) \quad (1)$$

of the roundtrip distance, with  $c$  denoting the speed of light.

The collected timestamps may be corrupted by noise and some of them could be altered by an attacker. Let  $m_i$  and  $o_i$  model the amount of fictional distance added due to the measurement noise and the attack respectively. An anchor misreporting  $t_{iS0}$  or  $t_{iR0}$  can lead to an  $o_i \neq 0$ . Likewise, the target can misreport  $t_{0Si}$  or  $t_{0Ri}$ , which again leads to  $o_i \neq 0$ . It holds that

$$r_i = d_i + m_i + o_i, \quad i \in \mathcal{I} \quad (2)$$

where  $d_i = \|\mathbf{x}_i - \mathbf{x}\|_2$ .

Moving  $o_i$  to the other side of (2) and expanding  $d_i$  yields

$$r_i - o_i = \|\mathbf{x}_i - \mathbf{x}\|_2 + m_i. \quad (3)$$

Squaring both sides of (3) and separating knowns from unknowns leads to

$$s_i = \|\mathbf{x}\|_2^2 - 2\mathbf{x}_i^T \mathbf{x} + u_i + n_i, \quad i \in \mathcal{I} \quad (4)$$

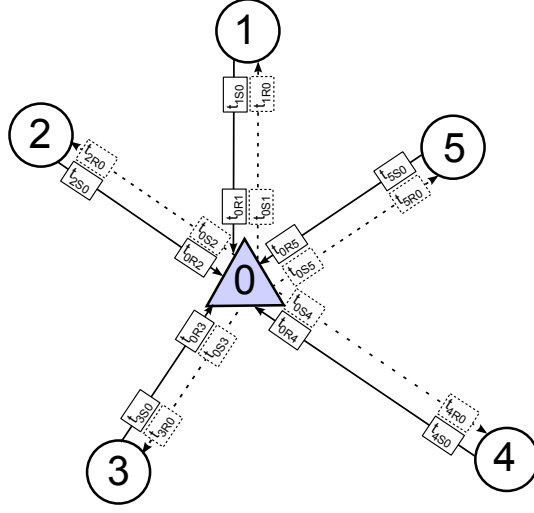


Figure 1: Schematic representation of the TWR ranging protocol. The circles represent anchors and the triangle is the target. Solid lines represent transmissions from an anchor to the target. Dashed lines denote transmissions from the target to the anchors.

where

$$\begin{aligned}
 s_i &= r_i^2 - \|\mathbf{x}_i\|_2^2 \\
 u_i &= 2r_i o_i - o_i^2 \\
 n_i &= 2d_i m_i + m_i^2.
 \end{aligned} \tag{5}$$

Equivalently, (4) can be rewritten in matrix form as  $\mathbf{s} = \mathbf{A}\mathbf{y} + \mathbf{u} + \mathbf{n}$ , where  $\mathbf{y} = [ \|\mathbf{x}\|_2^2, \mathbf{x}^T ]^T$

$$\begin{aligned}
 \mathbf{s} &= \begin{bmatrix} r_1^2 - \|\mathbf{x}_1\|_2^2 \\ r_2^2 - \|\mathbf{x}_2\|_2^2 \\ \vdots \\ r_M^2 - \|\mathbf{x}_M\|_2^2 \end{bmatrix}_{M \times 1}, \quad \mathbf{u} = \begin{bmatrix} 2r_1 o_1 - o_1^2 \\ 2r_2 o_2 - o_2^2 \\ \vdots \\ 2r_M o_M - o_M^2 \end{bmatrix}_{M \times 1}, \\
 \mathbf{n} &= \begin{bmatrix} 2d_1 m_1 + m_1^2 \\ 2d_2 m_2 + m_2^2 \\ \vdots \\ 2d_M m_M + m_M^2 \end{bmatrix}_{M \times 1}, \quad \mathbf{A} = 4 \begin{bmatrix} 1 & -2\mathbf{x}_1^T \\ 1 & -2\mathbf{x}_2^T \\ \vdots & \vdots \\ 1 & -2\mathbf{x}_M^T \end{bmatrix}_{M \times (p+1)}.
 \end{aligned} \tag{6}$$

A necessary condition to get a unique position estimate is  $\text{rank}(\mathbf{A}) = p + 1$ . Consequently  $M \geq p + 1$ . For  $p = 2$  it follows that  $M \geq 3$ . Additionally, it can be seen that a single misbehaving anchor can influence only one element of  $\mathbf{u}$ . A misbehaving target, however, can potentially influence any number of elements. When only a few anchors behave maliciously or the target misbehaves a few times, we can expect the vector  $\mathbf{u}$  to be sparse with a few nonzero elements.

### 3 Cost function and algorithm

The system model derived for the TWR protocol is the linear model

$$\mathbf{s} = \mathbf{A}\mathbf{y} + \mathbf{u} + \mathbf{n} \quad (7)$$

When the anchors and target behave reliably,  $\mathbf{u} = \mathbf{0}$  and a position estimation is commonly posed as minimizing  $J(\mathbf{y}) = \|\mathbf{s} - \mathbf{A}\mathbf{y}\|_2^2$ . This minimization problem admits the well known least squares (LS) solution  $\mathbf{y}^* = \mathbf{A}^\dagger \mathbf{s}$ , where  $\mathbf{A}^\dagger = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T$  is the Moore-Penrose pseudoinverse of  $\mathbf{A}$ . In the presence of outliers, however,  $\mathbf{u} \neq \mathbf{0}$ , and it is well known that the LS estimator can produce a very poor estimate [14, Ch. 1]. Even a single outlier can severely distort it.

A more robust alternative to the LS estimator is to minimize the cost function

$$J(\mathbf{y}, \mathbf{u}, \lambda) = \frac{1}{2} \|\mathbf{s} - \mathbf{A}\mathbf{y} - \mathbf{u}\|_2^2 + \lambda \|\mathbf{u}\|_1 \quad (8)$$

$J$  is essentially a sparse residuals formulation of the cost function. The  $\ell_1$ -norm favors a sparse  $\mathbf{u}$  and a  $\mathbf{y}$  that minimizes the  $\ell_2$ -norm of the noise vector.

A group coordinate descent algorithm can be used to solve  $J$  [10]. Alternatively, it can be transformed into a more traditional LASSO problem [9]. Very efficient solvers such as Least Angle Regression (LARS) [6] and GLMNET [8] can be then used.

The LASSO formulation of the problem is given by minimizing  $J$  over  $\mathbf{y}$  and plugging the solution back into  $J$ . The cost function then becomes

$$\begin{aligned} J_L(\mathbf{u}, \lambda) &= \|(\mathbf{I} - \mathbf{A}\mathbf{A}^\dagger)(\mathbf{s} - \mathbf{u})\|_2^2 + \lambda \|\mathbf{u}\|_1 \\ &= \|\mathbf{z} - \mathbf{B}\mathbf{u}\|_2^2 + \lambda \|\mathbf{u}\|_1 \end{aligned} \quad (9)$$

where  $\mathbf{B} = \mathbf{I} - \mathbf{A}\mathbf{A}^\dagger$  and  $\mathbf{z} = \mathbf{B}\mathbf{s}$ . The position estimate  $\mathbf{y}^*$  can be calculated as  $\mathbf{y}^* = \mathbf{A}^\dagger(\mathbf{s} - \mathbf{u}^*)$ , where  $\mathbf{u}^* = \arg \min_{\mathbf{u}} J_L$ .

Let  $(\mathbf{y}^*, \mathbf{u}^*)$  be the solution of  $\min_{(\mathbf{y}, \mathbf{u})} J$  for  $\lambda$  fixed and  $N_o$  the true number of outliers. As mentioned earlier, choosing the parameter  $\lambda$  controls the sparsity of  $\mathbf{u}^*$ . Choosing a good value for  $\lambda$  is crucial, as a very small value will tend to give many false positives, whereas a very big value will make  $\mathbf{u}$  very sparse and outliers might go undetected. The best value would be the one for which  $\|\mathbf{u}^*\|_0 = N_o$ .

A way to deal with this problem is to solve for multiple values of  $\lambda$  and to select a suitable set of solutions based on some prior knowledge, or estimation procedure [9]. For example, assume a grid of  $L$  values for  $\lambda$ , denoted by  $\lambda^{[k]}, k = 1, 2, \dots, L$ . Each value  $\lambda^{[k]}$  corresponds to a solution set  $(\mathbf{y}^{*[k]}, \mathbf{u}^{*[k]})$ ,  $k = 1, 2, \dots, L$ . Then,

- If  $N_o$  is known, a solution  $(\mathbf{y}^{*[j]}, \mathbf{u}^{*[j]})$  for which  $\|\mathbf{u}^{*[j]}\|_0 = N_o$  can be chosen.
- If the noise variance  $\sigma_n^2$  is known, a solution  $(\mathbf{y}^{*[j]}, \mathbf{u}^{*[j]})$  can be chosen, for which  $j = \arg \min_k (|\sigma_n^2 - \text{var}(\mathbf{s} - \mathbf{A}\mathbf{y}^{*[k]} - \mathbf{u}^{*[k]})|)$ .

### 4 Simulation

The performance of our method in estimating the correct position of the target was tested using the threat model proposed in [11]. The adversary attempts to contaminate anchors, so that their measurements point at location  $\mathbf{x}_a$ . The true location of the target,  $\mathbf{x}$  is at distance  $d_a$  from  $\mathbf{x}_a$ . The effect on the linear model is that those linear equations corresponding to contaminated measurements are satisfied by  $\mathbf{x}_a$ . The rest

are satisfied by  $\mathbf{x}$ . This can be viewed as some equations “voting” for  $\mathbf{x}_a$  as a solution and the rest “voting” for  $\mathbf{x}$ .

In the simulated scenarios, 30 anchors are randomly dispersed in a  $500 \times 500m^2$  rectangular area. The target is also dispersed randomly in the same area. A fusion center gathers measurements that are corrupted by Gaussian noise of variance  $\sigma_m^2$ ,  $m_i \sim \mathcal{N}(0, \sigma_m^2)$ . We examine the cases where either 10%, 20% or 40% of the anchors give wrong measurements. It is randomly determined which of the anchors will send contaminated measurements.

A linear TWR model is built. A block coordinate descent algorithm [10] is then used to provide the vectors  $\mathbf{u}^{*[k]}$  and  $\mathbf{y}^{*[k]}$ . The best candidates are chosen using prior knowledge of the noise variance, as described in Section 3.

The simulation results can be seen in Fig. 2. It can be seen that when the number of outliers is small, for example 10% or 20%, the root of the mean square error (RMSE) of the distance of the true location of the target from the estimated location is significantly better using a simple LS estimation. Since the LS estimation is not robust to outliers, the error grows as the strength of the attack grows. The robust estimation method gives comparable results to LS for low attack strengths, since in this case the outliers’ strength is comparable to the noise level and are treated as noise. This is not the case, however, for high attack strengths, where outliers are successfully detected by the robust estimator and we witness a decoupling of the RMSE from the attack strength, unlike the LS case. It can be seen on Fig. 2(a) that for small contamination rates, most outliers are successfully detected with a small number of false positives.

The situation is however different for high contamination rates, where many elements of  $\mathbf{u}$  are expected to be nonzero. If  $\mathbf{u}$  is not sparse enough, then the chances of getting good estimations for  $\mathbf{u}$  are significantly lower. This can be seen in the 40% contamination rate case. The RMSE is big, as is the percentage of false positives, an indication that the quality of the estimation will not be satisfactory when many outliers are present.

## 5 Conclusions

In this paper we have provided a linearized form of the localization problem when using the TWR protocol in UWB-IR WSNs. This linear model takes the presence of outliers into account and we have established that when only a few ranging measurements are corrupted by an attack, the vector  $\mathbf{u}$  that captures the influence of the outliers will be sparse. The linearity of the problem in combination with the outlier sparsity makes robust location estimation techniques such as LASSO and sparse residuals feasible. By using simulations we have shown that when only a few outliers are present, the RMSE will mostly depend on the measurement noise variance. When lots of outliers are present, however, this method performs in a less satisfactory manner, since LASSO and sparse residuals do not correctly identify the outliers when  $\mathbf{u}$  is not sparse.

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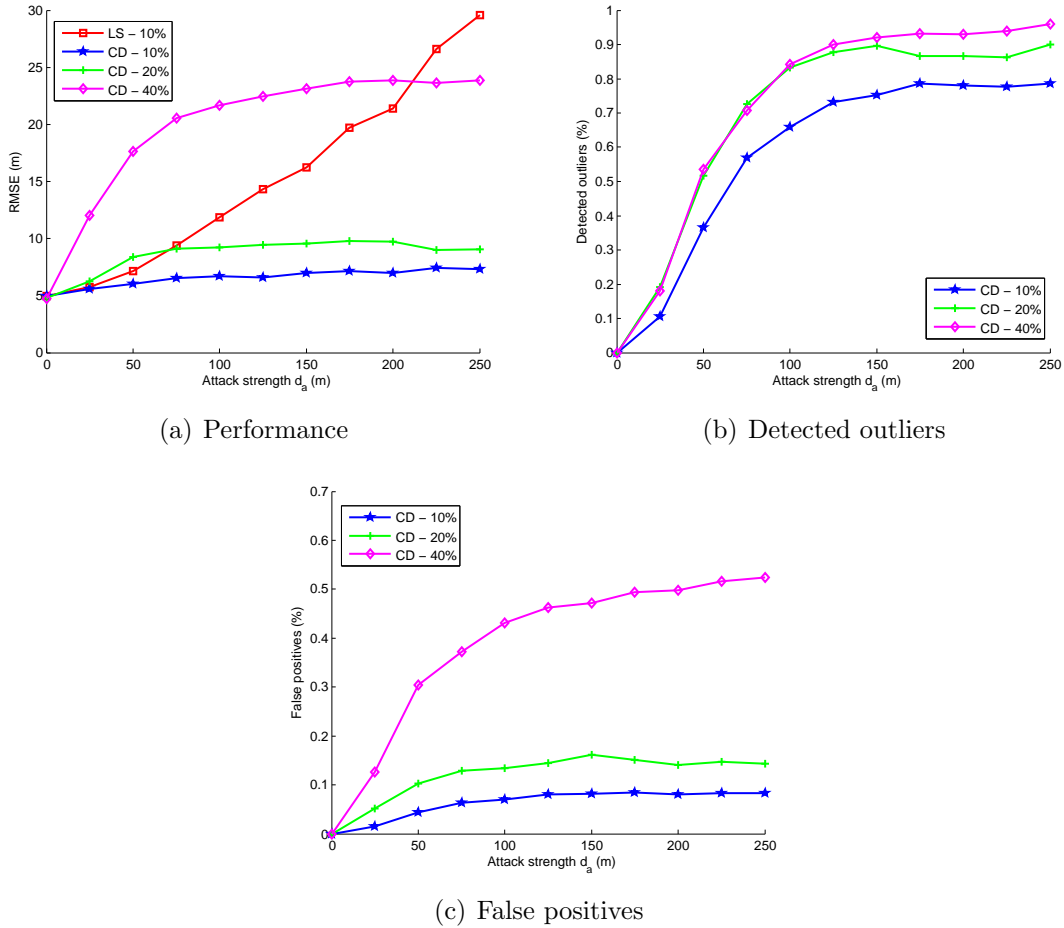


Figure 2: Performance of the method for  $\sigma_m^2 = 400m^2$  and contamination rates 10%, 20% and 40%. LS denotes the LS estimator results and CD the robust estimator results, calculated using the block coordinate descent algorithm. Each point is averaged over 500 trials.

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